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Prediction of turbulent heat transfer with surface blowing using a non-linear algebraic heat flux model

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Abstract

The paper reports on the prediction of the effects of blowing on the evolution of the thermal and velocity fields in a flat-plate turbulent boundary layer developing over a porous surface. Closure of the time-averaged equations governing the transport of momentum and thermal energy is achieved using a complete Reynolds-stress transport model for the turbulent stresses and a non-linear, algebraic and explicit model for the turbulent heat fluxes. The latter model accounts explicitly for the dependence of the turbulent heat fluxes on the gradients of mean velocity. Results are reported for the case of a heated boundary layer which is first developed into equilibrium over a smooth impervious wall before encountering a porous section through which cooler fluid is continuously injected. Comparisons are made with LDA measurements for an injection rate of 1%. The reduction of the wall shear stress with increase in injection rate is obtained in the calculations, and the computed rates of heat transfer between the hot flow and the wall are found to agree well with the published data.

Keywords: Turbulent boundary layer; Blowing; Rational heat flux model; Thermal stress reduction

1. Introduction

This paper is concerned with the prediction of the thermal and velocity fields associated with the development of a flat-plate boundary layer in conditions of continuous blowing from a porous surface. There are many instances where such flows are of practical interest. In aeronautics, surface blowing is used to modify the boundary-layer flow over lifting surfaces. In the case of combustion chamber or turbine blades, there is a need to protect those walls that are subjected to high thermal stresses. There are a number of alternative techniques that can be used to protect walls from the hot fluxes. Those include such measures as film cooling, impingement or blowing. The latter technique involves the injection of cold gas through a porous section of the protected plate. Numerous studies have been conducted to understand the interactions between the main and the injected flows. The modifications of the dynamical boundary layers have received much attention and we can note in this context the experiments carried out by Romanenko and Kharchenko (1963), Moffat and Kays (1968), Simpson et al. (1969), Baker and Launder (1974) and the theoretical analysis of Simpson (1970), Silva-Freire (1988), Thomas and Kadry (1990). In contrast, few studies have been conducted for the case where the temperatures of the main stream and the injected flow were different. Recently, numerical and experimental works have been conducted by Campolina Franca et al. (1996), Bellettre et al. (1997, 1999a,b) to study the influence of blowing through a porous plate on the dynamical and thermal boundary layers. The numerical approach utilised a model of the blowing and a relatively simple turbulence model for the main flow. In this paper, we investigate the modification of the boundary layer due to blowing using more advanced models to account for the effects of turbulence on the transport of momentum and thermal energy. In the first part of this paper, the model of blowing through porous matrix and

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Nomenclature

C_f friction factor $\left(\frac{C_f}{2} = \frac{\tau_w}{\rho_e U_e^2}\right)$
d boundary layer thickness
F injection rate $\left(F = \frac{(\rho U_2)_w}{(\rho U_1)_e}\right)$
h convective heat transfer coefficient ($h = 10$ W/m^2 K)
k turbulent kinetic energy (m^2/s^2)
Re_{x1} Reynolds number $\left(Re = \frac{\rho U_{1e} x_1}{\mu}\right)$
St Stanton number $\left(St = \frac{h}{\rho U_{1e} c_p}\right)$
T temperature (K)
U velocity (mean value) (m/s)
U^* friction velocity (m/s) $\left(U^* = \sqrt{\tau_w/\rho}\right)$
$\overline{u_i \theta}$ turbulent heat fluxes (mK/s)
x spatial coordinate (m)
y^+ dimensionless coordinate $\left(y^+ = \frac{\rho U^* x_2}{\mu}\right)$

Greeks
Δ enthalpy thickness (m) $\Delta = \int_0^\infty \frac{\rho U_1}{\rho_e U_{1e}} \frac{T - T_e}{T_w - T_e} \mathrm{d}x_2$
ε turbulent kinetic energy dissipation rate (m^2/s^3)
φ porosity
μ dynamic viscosity (kg/m s)
Θ mean value of the transported scalar (i.e. enthalpy)
θ momentum thickness (m) $\left(\theta = \int_0^\infty \frac{\rho U_1}{\rho_e U_{1e}} \left(1 - \frac{U_1}{U_{1e}}\right) \mathrm{d}x_2\right)$
ρ density (kg/m^3)
τ shear stress (Pa)

Subscripts
o without blowing
1 longitudinal direction
2 vertical direction
e main flow
inj injected (at pore exit)
t turbulent
w wall

turbulence model will be presented. Following that, results for the boundary layer velocity profiles, friction factors and Stanton numbers will be presented and compared to reported experimental data.

2. The models used

2.1. Model for blowing through a porous plate

The model used here to represent blowing through the porous section takes directly into account the influence of the injected fluid on the boundary layer. This has the advantage of not requiring modification of the basic equations of the main flow and, also, of being valid to most configurations of practical interest. The porous plate is modelled as a succession of solid strips and open holes through which fluid is injected at the appropriate rate. The proportion of the two is determined according to the desired wall porosity. Details on the effect of the porosity and the pore size can be found in Bellettre et al. (1999a). The effects of the shear stress exerted by the solid strips on the main flow due to are taken into account directly via the logarithmic law of the wall for momentum. Similarly, the heat transfer rate between the solid strips and the main flow is governed by the law of the wall for thermal energy (Bellettre et al., 1999a). The effect of injection is directly taken into account by mass, momentum and thermal energy brought by the cold fluid from the holes. This treatment constitutes a discrete model of surface blowing is embedded in the Navier Stokes code used for predicting the external turbulent boundary layer flow. As high turbulence Reynolds number forms of the Reynolds-stress and heat flux models are used, integration is carried out to a location down to y^+ of about 30 where the numerical solutions were matched to the universal log-law with coefficients set to values appropriate for a flat-plate boundary layer in zero pressure gradient.

2.2. Model of turbulence

In previous studies (Bellettre et al., 1997, 1999a,b), the RNG k–ε model was used to model the turbulent stresses. It was found that this model was inadequate at taking into account the effects of blowing on the various turbulence parameters, even if the mean flow was well predicted. Furthermore, ad hoc correlations were required in order to obtain estimates of the heat fluxes at the wall. In the present study, we use the model of Younis et al. (1996) to determine the turbulent heat fluxes while closure of the time-averaged streamwise momentum equation is achieved using a complete Reynolds-stress-transport model of turbulence. Details of this model may be found in Gibson and Younis (1986a,b). Briefly, turbulent transport is modelled following Daly and Harlow's (1970) simple gradient-transport proposal, viscous dissipation is assumed to be isotropic and the rate of dissipation of turbulence kinetic energy obtained from the solution of the standard equation. The difficult pressure–strain correlations are obtained following the proposals of Launder et al.

(1975) using the simpler of the two models for the rapid part proposed in that paper. As explained earlier, the porous wall was treated as a succession of solid and open sections and for the regions of flow directly above the solid sections, the modifying effects on the turbulence field are included as proposed by Gibson and Launder (1978). Analysis, using 'Tensor Representation Theory', produced an explicit and algebraic model for the turbulent heat fluxes that made these quantities dependent on the Reynolds stresses and on the gradients of mean velocity. Such dependence is present in the exact equations for the turbulent heat fluxes but is absent from conventional models such as Fourier's Law. The model proposed by Younis et al. (1996) takes the form:

$$-\overline{u_i\theta} = C_1\frac{k^2}{\varepsilon}\frac{\partial\Theta}{\partial x_i} + C_2\frac{k}{\varepsilon}\overline{u_iu_j}\frac{\partial\Theta}{\partial x_j} + C_3\frac{k^3}{\varepsilon^2}\frac{\partial U_i}{\partial x_j}\frac{\partial\Theta}{\partial x_j} + C_4\frac{k^2}{\varepsilon^2}\left(\overline{u_iu_k}\frac{\partial U_j}{\partial x_k} + \overline{u_ju_k}\frac{\partial U_i}{\partial x_k}\right)\frac{\partial\Theta}{\partial x_j} \quad (1)$$

There are four constants that were determined in the original reference by matching the model to data from large Eddy simulations of homogeneous turbulence with uniform strain rate and constant scalar gradients. These values are given as: $(C_1, C_2, C_3, C_4) = (-0.0455, 0.373, -0.00373, -0.0235)$.

As this model has been developed for the case of constant fluid properties, only the case of small temperature differences are examined here.

3. Results

The models for discrete surface blowing and for the turbulent scalar heat flux were implemented using a numerical code solving the dynamic and thermal equations of the boundary layer. The code solves the two-dimensional forms of the governing equations in marching integration, starting with assumed initial distributions for the mean velocity and the Reynolds stresses. Grid-independence checks were performed and numerically-accurate results were typically obtained with 50 nodes unevenly distributed in the cross-stream direction. The forward-step size was limited to 2% of the local boundary-layer width. In the injection region over the porous plate, this quantity was reduced by an order of magnitude to only 0.2% of the local width. A sketch of the geometry under consideration is presented in Fig. 1. The main flow attains equilibrium on the impermeable wall before being subjected to surface blowing. The temperature of the main flow is 45 °C whereas the boundary condition imposed on the impermeable plate is external convection with the surrounding medium at a temperature of 20 °C. The Reynolds number is equal to 8.15×10^5 before the start of injection. The temperatures of the coolant and the porous matrix were fixed at 31 °C,

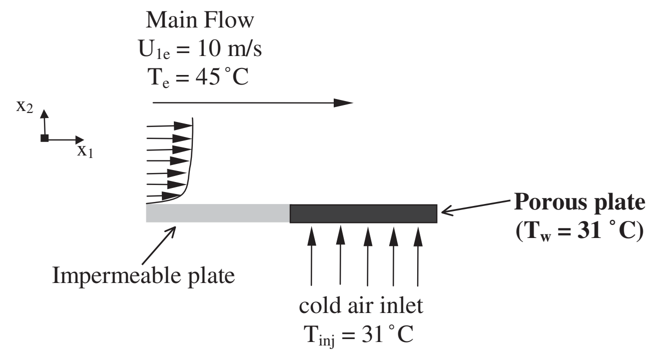

Fig. 1. Geometry and coordinates of problem considered.

in accord with the experimental set-up. The injection rate, F, is defined as:

$$F = \frac{\rho_w U_{2w}}{(\rho U_1)_e} \quad (2)$$

where U_{2w} is the average value of the vertical velocity at the wall.

The effect of the wall porosity is taken into account in the velocity imposed at the exit of the holes: $U_{inj} = U_{2w}/\varphi$. In the present case, the wall porosity is 1/3, corresponding to experimental set-up.

In Fig. 2, the effect of the blowing on the longitudinal velocity profile can be observed. Results are plotted at a streamwise location corresponding to $Re_{x1} = 10^6$. The boundary layer thickness is substantially increased with the injection of additional mass into the main flow. In Fig. 3, the predicted profiles of mean velocity for the $F = 1\%$ case are compared with the LDA measurements of Rodet et al. (1998). Clearly the extent of agreement between the Reynolds-stress model predictions and the data is quite satisfactory.

Blowing through a porous matrix has long been recognised as an effective technique for viscous drag reduction. In Fig. 4, the predicted streamwise development of the skin-friction coefficient, C_f, is presented for five different values of surface injection rates. For the no

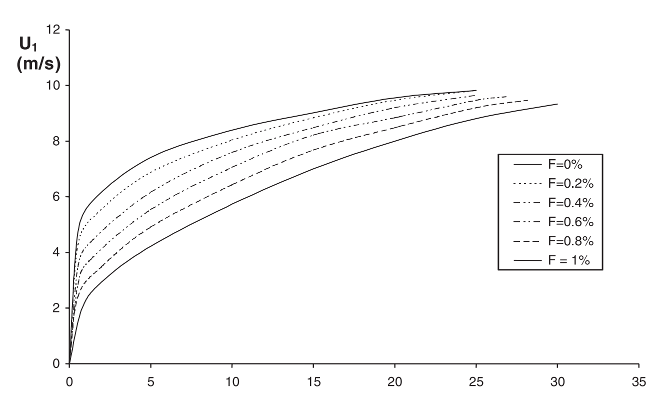

Fig. 2. Predicted equilibrium velocity profiles as a function of F.

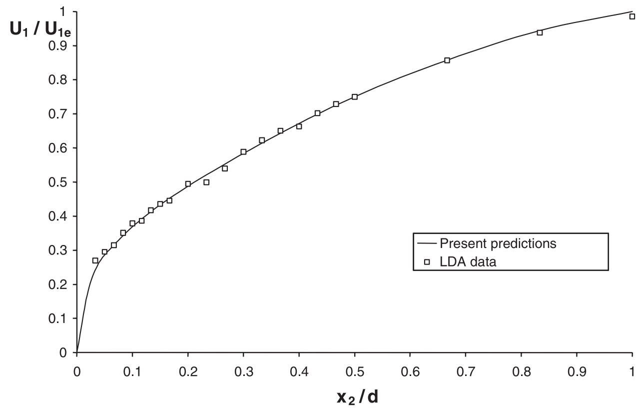

Fig. 3. Comparison of the present results with the LDA data of Rodet et al. (1998).

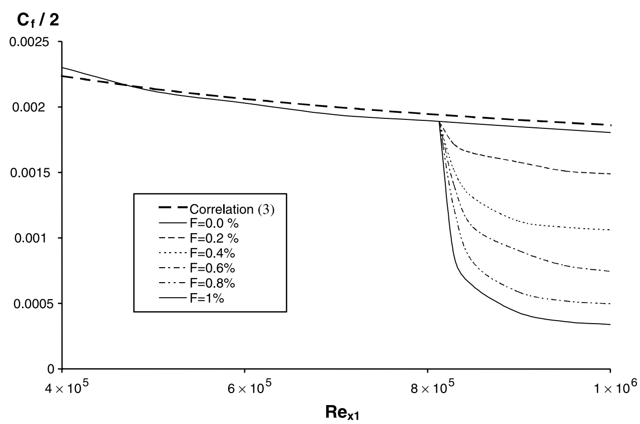

Fig. 4. Predicted variation of skin-friction coefficient with Reynolds number for $0 < F < 1\%$.

blowing case, the numerical results are compared with the correlations reported by Blasius (3):

$$\frac{C_{\mathrm{fo}}}{2} = 0.0295 Re_{x1}^{-0.2} \tag{3}$$

The differences between predictions and correlations for the $F = 0\%$ case are very small. When blowing occurs, the friction factor is dramatically reduced, even for weak injection rates. This reduction becomes very high when blowing increases. For example, for a 1% injection rate, the friction factor is reduced by a factor 4 (at $Re_{x1} = 10^6$). It can also be noted that this reduction is very rapid and occurs as soon as the blowing is applied. Comparisons with experimental results of Baker and Launder (1974) are done at the end of the porous plate. The results are plotted in Fig. 5. These comparisons can be performed because they correspond to similar situations: injection rate and momentum thickness, θ, are equal in each case. In fact, Simpson et al. (1969) have shown that local friction factor depends only of these two parameters. Furthermore, Fig. 5 presents the validation of our numerical simulation as our results are in good agreement with the data from Baker and Launder.

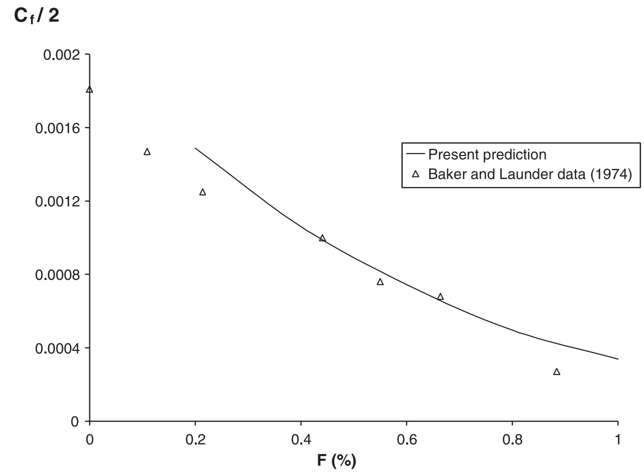

Fig. 5. Predicted and measured variation of friction factors with F.

Temperature profiles above the porous plate can be seen in Fig. 6. The injection rate increases from 0% to 1%. We can note the decrease of the near wall temperature and the important deviating effect of the blowing. It implies an important decrease of the heat flux received by the plate when F increases. The Stanton number calculations will confirm this reduction.

Stanton numbers, obtained at $Re_{x1} = 10^6$, are compared to Stanton numbers from the published literature (Fig. 7). The data correspond to a similar situation, in terms of injection rate and enthalpy thickness, Δ. In fact, Whitten et al. (1970) have shown that the local values of Stanton numbers depend only on these two parameters (as Simpson et al. (1969) have shown to also be the case for C_{f}). Again good agreement is obtained between the present results and the published measurements indicating the validity of the models employed for both the dynamic and thermal aspects of these difficult flows.

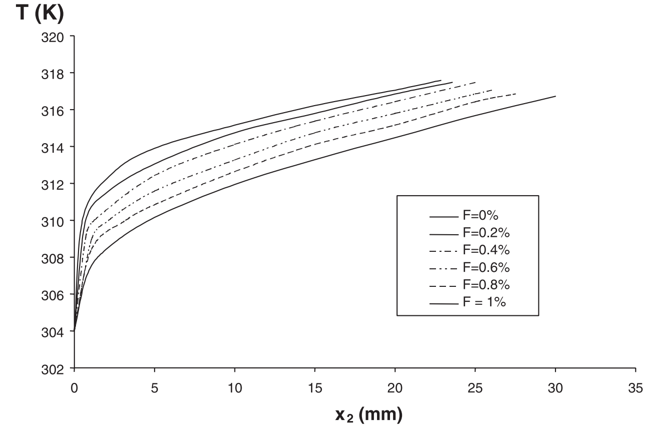

Fig. 6. Effect of blowing on the temperature profile.

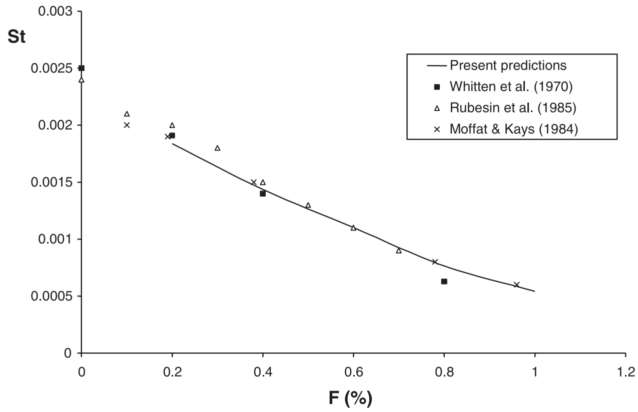

Fig. 7. Predicted and measured (Whitten et al., 1970; Rubesin et al., 1985; Moffat and Kays, 1984) variation of Stanton number with F.

4. Conclusions

The development of the turbulent boundary layer on a flat plate subjected to surface blowing has been studied numerically. The model for discrete surface blowing of Bellettre et al. (1999a) was used in conjunction with a complete Reynolds-stress transport model of turbulence and the non-linear algebraic model for the turbulent heat fluxes of Younis et al. (1996). The computational results were compared with existing measurements. Concerning the mean flow characteristics, both the dynamic (velocity profiles, friction factors) and thermal (temperature profiles, Stanton numbers) aspects have been validated. Attention will now turn to the details of turbulent transport as characterised by the velocity and temperature fluctuations in the boundary layer. Appropriate experimental facilities (Bataille et al., 1999, 2001) are now available for obtaining measurements in the presence of temperature gradients. These will permit the validation of the turbulent quantities obtained by the simulations. The extension of the study to cases of high temperature gradients will then be considered by taking into account the temperature dependence of the fluid properties in the model.

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